NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2498

FLIGHT INVESTIGATION OF THE EFFECT OF ATMOSPHERIC TURBULENCE ON THE CLIMB PERFORMANCE OF AN AIRPLANE

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SUMMARY

The results of an investigation consisting of a series of one-engine climb tests with a twin-engine transport airplane indicate that light turbulence of the type generally present in clear air over flat terrain has no significant effect on the average rate of climb for a series of runs. Turbulence does, however, increase the variation in the rate of climb from run to run. The standard deviation of the rate of climb between runs attributable to turbulence decreases rapidly when the climb duration is increased from 1 to 5 minutes. The effects of atmospheric turbulence on the variations in the rate of climb appeared to be largely independent of the center-of-gravity location.

INTRODUCTION

Atmospheric turbulence is one of the factors which affect the realizable climb performance of an airplane in flight. Although the effects of atmospheric turbulence on the realizable performance of an airplane in flight have been the subject of a number of investigations, the effects are as yet neither known nor understood. The results of various investigations appear to be inconclusive and, in some cases, contradictory. Experimental results reported in reference 1, for example, indicate that an appreciable reduction in airplane performance results from the action of atmospheric turbulence. On the other hand, a theoretical analysis presented in reference 2 suggests that the effects of atmospheric turbulence on airplane performance are on the whole small and act to increase the realizable performance. The validity of the results of reference 1 has been questioned because the large difference in altitude between the smooth- and rough-air tests necessitated large corrections in the reductions of the data. The theoretical analysis of reference 2 necessarily suffers from various simplifications, both in the definition of atmospheric turbulence and in the analysis of airplane motions in a turbulent stream. Because of these limitations, further investigation of this problem seemed desirable.

In order to determine some of the effects of turbulence on climb performance, a series of flight tests in smooth air and under conditions of light turbulence with a twin-engine transport airplane have been completed by the National Advisory Committee for Aeronautics. Primary interest in the effects of atmospheric turbulence on climb performance is concerned with the condition of reduced power associated with an engine failure where the further loss of ability to climb may result in a dangerous condition or an accident. In order to make the results directly applicable to the one-engine-inoperative condition on a multiengine airplane by including the effects associated with reduced and asymmetrical power, the present tests were restricted to the one-engine-inoperative condition. Since the effects of turbulence might vary with airplane longitudinal stability, flight tests were made at two center-of-gravity locations. This paper summarizes the primary results obtained in regard to the effect of turbulence on the average rate of climb and on the variations in the rate of climb between separate climbs. Some effort is also made to investigate the characteristics of the airplane flight path that give rise to the variations in the rates of climb.

TEST PROCEDURES AND CONDITIONS

Test flights were made with a twin-engine transport airplane. The airplane characteristics are summarized in table I. A torquemeter was installed on the operative engine in order to provide a measure of the actual power output. Standard NACA recording instruments were used to measure airspeed, static pressure, normal acceleration, free-air temperature, torque pressure, and rotational speed. The records were synchronized with an NACA timer which marked the records at $\frac{1}{2}$- second intervals.

The airplane weight was controlled as closely as operationally possible and averaged about 24,200 pounds. The weights for individual runs, however, were sometimes several hundred pounds above or below the average. The climb tests were made at an indicated airspeed of 97 miles per hour (the take-off safety speed for this airplane); this speed was chosen because it was close to the best angle of climb. The airplane configuration was clean for all runs; that is, the flaps and landing gear were up. In order to determine the effects of longitudinal stability on the climb performance in rough air, a forward center-of-gravity position (12.9 percent M.A.C.) was used for a set of eight flights and a rearward center-of-gravity position (26.3 percent M.A.C.) was used for another set of eight flights. The left engine was feathered for all runs and about 950 horsepower at 2,550 rpm was used on the right engine. This power was chosen to minimize effects due to high power settings and to provide a low rate of climb in order to restrict the altitude range of the tests.

The pilots were instructed to attempt to hold the airplane in a constant climb attitude by flying the airplane with wings level and no change in heading. They were cautioned to use gradual stick motions in order to correct minor deviations in attitude or airspeed.

Each flight consisted of from four to seven climb runs each of about 5-minute duration. Climb runs were made over level terrain from about 2,000 to 3,000 feet above terrain. No strong efforts were made to make flights under conditions of severe turbulence, flight days being primarily determined by the availability of the airplane. Test runs were made in clear air during the morning and afternoon hours. The turbulence encountered was in general light and is considered representative of the experience of normal transport operations under similar conditions of terrain, weather, and season. The maximum effective gust velocity encountered during these tests was about 10 feet per second.

ANALYSIS AND RESULTS

The observed rates of climb for each run were corrected to geometric rates of climb, an average weight of 24,200 pounds, a standard altitude density of 4,000 feet (the average for the present tests), and 950 horsepower in accordance with the performance reduction methods outlined in appendix A. No corrections were necessary for speed changes because the beginning and end of a run were purposely selected to insure no airspeed change. Table II presents the rates of climb and the run duration for each run.

The selection of an appropriate scale for atmospheric turbulence is somewhat arbitrary. Two types of turbulence may be important; the long wave movements which tend to lift or drop the airplane as a free particle and the turbulence of the scale of the airplane size which results in normal accelerations and rotations of the airplane which may introduce aerodynamic effects on the airplane lift or drag. The presence of one scale of turbulence is probably generally associated with the existence of the other scale of turbulence. Inasmuch as normal accelerations have been used sucessfully as a measure of turbulence in regard to structural studies, they were also used in the present study. Although the normal accelerations are a function of the airplane airspeed, the use of the acceleration data directly was permissible in the present case since the airspeed for all runs was held constant.

The acceleration experience in rough air generally consisted in a series of acceleration increments of various intensities which can be summarized in the form of a frequency distribution. The frequency distribution of acceleration increments for each run was consequently evaluated and the distributions are summarized in tables II(a) and II(b) for the forward and rearward center-of-gravity positions, respectively.

Consideration of the frequency distributions of normal acceleration experienced on each of the runs indicated that available test data could be divided into three classes of turbulence intensity. Although the particular scale chosen was arbitrary, the separation used gave a roughly equal number of runs for each class of turbulence intensity. The particular classification used is as follows:

Class	Intensity	Description
I	Smooth	No acceleration increments equal to or greater than 0.10g
II	Intermediate	More than zero but less than five acceleration increments per minute greater than 0.10g
III	Rough	More than five acceleration increments per minute greater than 0.10g

The turbulence class of each run is also given in table II.

The mean rates of climb and the standard deviations (reference 3) were computed for each degree of turbulence intensity and each center-of-gravity position and are summarized in table III. It will be noted that, if the frequency distribution is assumed to be a normal distribution, it is completely specified by the two parameters, the mean and the standard deviation. As a further statistic, the standard deviation for the mean value is also given for each test condition in table III. This value represents a measure of the reliability of the mean value. The mean rates of climb and the standard deviations as a function of turbulence intensity are shown in figures 1 and 2, respectively, for both the forward- and rearward-center-of-gravity tests. The relative frequency or probability of a value of rate of climb falling below given values is shown in figure 3 for both smooth and rough air. The curves shown were obtained by fitting normal distributions to the observed data in accordance with the methods of reference 3.

In order to investigate the nature of the airplane climb in greater detail and to obtain a better insight into the characteristics of the effects of atmospheric turbulence on the airplane flight path, further analysis was considered to be desirable. It has been suggested that the nature of turbulence action on the airplane flight path is such as to cause relatively short period oscillations about some mean flight path. Under this concept, the standard deviation of the rate of climb would be expected to decrease as a direct function of the duration of the climb. Since this point is of some importance in connection with present international deliberations on climb standards, some effort was made to examine the nature of the airplane oscillations about an average path.

From each test run an arbitrary period covering 4 minutes was selected. These 4-minute runs were separated into four individual minutes and the rate of climb, corrected for speed change (appendix A), for each minute was noted. These data are summarized in table IV(a) for the forward-center-of-gravity test flights and in table IV(b) for the rearward-center-of-gravity test flights. The intensity of turbulence for each minute was represented by the turbulence classification for the entire run. A check indicated that the classification of turbulence intensity for the individual minutes of each run on the basis of the turbulence-intensity classification previously outlined did not materially affect the results obtained. Average rates of climb for 2-minute periods were obtained by averaging the values for the first and second minutes and for the third and fourth minutes. Mean values, standard deviations, and standard deviations of the mean were obtained for each of the 1-minute, the 2-minute, and the entire 4-minute runs. These data are summarized in table V. Figure 4 summarizes the results obtained for the variation in the standard deviation of the rate of climb as a function of the duration of run.

PRECISION OF RESULTS

A large number of sources of error are present in the measurement of the rate of climb. These errors arise from instrument inaccuracies, record-reading inaccuracies, and the approximate nature of the performance reduction methods. For the most part these errors are consistent and affect the absolute values of the rate of climb. For the present investigation, in which comparative rates of climb under various conditions are of interest, most of these errors have little influence on the results. The primary sources of the random errors affecting the present results are believed to arise from the errors in the determination of the height change and the inaccuracies of the reduction methods used. The precision within which the rate of climb can be determined is also a function of run duration inasmuch as the errors involved in determining the altitude change and in the reduction methods are largely of an absolute nature. Thus, for runs of longer duration the errors in the rate of climb will be averaged over time and reduced.

The climb reduction procedure used herein resulted in adjustments to the observed rates of climb which were, in general, small and only rarely exceeded a total correction value of 25 feet per minute. Thus, allowing a total error in reduction methods of 20 percent gives errors in the correction factors generally below 5 feet per minute. From considerations of the errors in the determination of the height change and in the reduction method, it is estimated that the values of the rate of climb obtained for individual runs are reliable to within ±10 feet per minute for the 5-minute runs and to as high as 25 feet per minute for the shortest run durations of 1 minute.

The variations in climb performance under smooth-air conditions noted in the present results are perhaps the best measure of the overall test precision. These variations which form an integral part of the present analysis, as will be noted in the discussion of the present results, in general substantiate the estimates of precision given herein.

DISCUSSION

Mean rate of climb.- Consideration of the results shown in figure 1 indicates no consistent change in the mean rate of climb with turbulence intensity for the flights either with the forward or with the rearward center-of-gravity position. Although small differences in the average rates of climb are evident, statistical criteria indicate that, because of the small samples, the observed differences cannot be considered significant but may be due to chance. The flight tests at the rearward center-of-gravity position yield rates of climb consistently greater than those at the forward position, the differences varying from 7 to 25 feet per minute. These differences because of their consistency are statistically significant and are apparently the result of a decrease in induced drag associated with the reduced wing lift and a smaller negative tail load with the rearward movement of the center-of-gravity position. Calculations indicated that the differences in the rate of climb obtained in these tests are reasonable.

Standard deviation of the rate of climb.- Examination of figure 2 indicates that the standard deviations of the rate of climb increase from smooth to rough air for both the forward and rearward center-of-gravity positions. The standard deviation of the rate of climb increases from about 14 feet per minute in smooth air to about 27 and 32 feet per minute in rough air. This result is statistically significant and indicates that appreciably more scatter in the rate of climb may be anticipated under conditions of turbulence than in smooth air. The figure also indicates that the variations in the rate of climb are somewhat greater for the forward-center-of-gravity tests than for the rearward-center-of-gravity tests; however, the differences between smooth and rough air are small and are not statistically significant. It may therefore be concluded that the effects of turbulence on the variations in the rate of climb are largely independent of the center-of-gravity position.

In view of the equivalence of the mean rate of climb in smooth and rough air and the greater variation of rate of climb in rough air, the realizable climb performance in rough air will fall below critical values below the mean with a greater frequency in rough air than in smooth air. Figure 3, which shows the probability of the rate of climb falling below given values for the present test conditions, indicates that low values of climb performance will occur with a far greater probability in rough air than in smooth air. For example, the results indicate that a rate of climb about 20 feet per minute below the mean value (140 ft/min) will occur roughly 3 times as frequently in rough air as in smooth air for the forward-center-of-gravity results. For the rearward-center-of-gravity tests, a rate of climb roughly 20 feet per minute below the mean (155 ft/min) occurs about 6 times as frequently in rough air as in smooth air.

The contributions of turbulence to the variations in the rate of climb may be separated from the contributions due to other causes by using the theory of errors. This separation may be effected by using the variances, the squares of the standard deviations. For the present results, the variance of the rates of climb consists of the variance associated with the basic precision of the tests and the contributions associated with the effects of turbulence. The variance of the smooth-air test results may be considered to represent a measure of the test precision. The variance of the rates of climb in smooth air may be attributed to errors in altitude, airspeed, and temperature measurements, wind-gradient effects, errors in weight estimation, minor deviations in torque pressure, rotational speed, and manifold pressure, and inaccuracies in the performance reduction methods and piloting influences. The variance of the rates of climb in rough air arises from all the factors associated with the smooth-air condition plus the effects of turbulence which include both the translatory and rotational motions of the airplane and the effects of pilot and turbulence interactions.

From these considerations, the contribution to the variance due to turbulence alone σ_t^2 may be obtained by the relation

$$\sigma_t^2 = \sigma^2 - \sigma_s^2$$

where

σ total standard deviation of rates of climb obtained in rough air (class III)

σ_s standard deviation of rates of climb obtained in smooth air or the test precision

If the foregoing relation between the variances is used, the following values of σ_t are obtained from the data of table III:

σ_t for forward center of gravity, feet per minute 29.5

σ_t for rearward center of gravity, feet per minute 24.0

These results indicate that for runs of about 5 minutes, turbulence of the type represented in the present tests introduces a standard deviation to the rate of climb of about 27 feet per minute.

Effects of climb duration.- Examination of figure 4 indicates that the standard deviation of the rate of climb decreases consistently for all test conditions when the duration of run is increased. This result is to some extent a consequence of the greater precision in testing that may be obtained for the average rates of climb in runs of long duration but is also associated with the characteristics of turbulence effect on the rate of climb. For smooth air (class I), the standard deviation of the rate of climb decreases from about 27 to 14 feet per minute for the tests at both center-of-gravity positions when the climb duration increases from 1 minute to 4 minutes. For the rough-air tests (class III), the standard deviation of rate of climb decreases from 85 feet per minute for the 1-minute runs to 36 feet per minute for the 4-minute runs for the tests at the forward center-of-gravity position. For the tests at the rearward center-of-gravity position the decrease is from 65 feet per minute for the 1-minute runs to 33 feet per minute for the 4-minute runs.

If the relationship derived previously for the standard deviation attributable to turbulence is used, values of σ_t can be derived from the available data for the runs of 1 minute, 2 minutes, and 4 minutes for both the forward-center-of-gravity tests and the rearward-center-of-gravity tests. The results obtained are summarized in figure 5. In addition, the values of the standard deviations of the rate of climb attributable to turbulence σ_t for entire runs (about 5 min) derived previously are shown in the figure. The figure thus presents the net standard deviation in the rate of climb that may be attributed to turbulent conditions as a function of run duration. Simple analytical considerations (appendix B) suggest that σ_t should vary inversely with the square root of run duration. Curves of the form

$$\sigma = Kt^{-1/2}$$

where K is a constant and t is the run duration measured in minutes, were consequently fitted to the data of figure 5 by the method of least squares. The results obtained are also shown in the figure.

Consideration of the standard deviations of the rate of climb due to turbulence shown in figure 5 indicates that for short periods, such as 1 minute, the standard deviation in the rate of climb is very large, roughly 60 and 80 feet per minute for the present tests. Inasmuch as these large values represent a sizable proportion of the one-engine-inoperative climb potential of modern transport airplanes, their consideration in the development of climb performance standards appears to be warranted. Fortunately, the standard deviation of the rate of climb decreases rapidly with run duration and is roughly 24 and 30 feet per minute for 5-minute runs. Consequently, for long periods of climb clear of terrain obstacles, the effect of turbulence of the type represented in the present tests on the rate of climb may be considered small and perhaps negligible.

Examination of figure 5 indicates that, for the data obtained from the rearward-center-of-gravity tests, the fitted curve is in good agreement with the data. For the forward-center-of-gravity test results, the agreement between data points and fitted curve is not so good, the fitted curve yielding appreciably higher values for σ_t for the 4- and 5-minute runs than the observed data. It does, however, appear reasonable to assume that the square-root relation yields an adequate approximation to the relation.

Implications.- The present results indicate the nature and order of magnitude of the effect of turbulence of the type represented in the present tests on the realizable performance of an airplane in rough air. Inasmuch as the one-engine-inoperative climb performance of transports (especially in civil aviation) is at least several hundred feet per minute turbulence of the type investigated will, under most conditions, have a small effect on the performance realized. For flight stages of short duration, however, such as those associated with the take-off and approach conditions and in particular for flight over terrain obstacles, the effects of turbulence as indicated by the present tests may be of sufficient magnitude to be critical.

The extrapolation of the present results to more severe turbulent conditions is somewhat conjectural. For turbulence of the same type but of greater intensity, the effects on the mean rate of climb may be expected to be small while the variations in the rate of climb may be expected to be larger than the values obtained in the present investigation. The effects of turbulence on climb performance associated with the more violent atmospheric motions present in such phenomena as thunderstorms, air-mass frontal storms, or flow over rough or sloping terrain cannot be inferred from the present results. The effects of air motions on the realizable performance in these cases are probably of far greater order of magnitude. Fortunately, these atmospheric conditions are encountered relatively infrequently and are therefore not so pertinent to the reduced-power case of the one-engine-inoperative condition.

CONCLUSIONS

The analysis of data obtained from a series of single-engine climb tests with a twin-engine transport airplane in smooth air and light turbulence has indicated the following results:

1. The average rate of climb for a series of climbs is unaffected by turbulence of the type considered.

2. The standard deviation of the rate of climb for the test runs was significantly greater in rough air than in smooth air; this result indicates an appreciable effect of turbulence on the variations in performance between climb runs.

3. The standard deviation of the rate of climb between runs decreases rapidly when the climb duration is increased from 1 to 5 minutes. The standard deviation of the rate of climb attributable to turbulence appears to vary inversely with the duration of the climb.

4. The effects of atmospheric turbulence on the variations in the rate of climb appeared to be largely independent of the center-of-gravity position.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
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APPENDIX A

CLIMB-PERFORMANCE REDUCTION EQUATIONS

Geometric Rate of Climb

The geometric rate of climb was obtained by the following relation:

$$R = \frac{\Delta h_p}{\Delta t} \frac{T}{T_s} \tag{A1}$$

where

R geometric rate of climb, ft/min

Δh_p change in pressure altitude obtained from altimeter, ft

Δt time duration of run, min

T absolute average temperature during run, $^{\circ}$F

T_s standard absolute temperature at average altitude of run, $^{\circ}$F

Corrections

Inasmuch as the corrections involved only minor departures from the desired constant test conditions, approximate corrections were considered adequate. Corrections to the measured rates of climb were made for the following conditions which were assumed to be standard for the present tests:

(1) A fixed power of 950 hp

(2) A fixed weight of 24,200 lb

(3) A fixed standard density altitude of 4,000 ft

For the 1-minute-climb data, a correction was also made for loss or gain of kinetic energy due to changes in airspeed from beginning to end of run.

The following relations were used in the performance reductions.

Correction for power.- The rate of climb may be expressed as

$$R = \frac{33000\eta B}{W} - \frac{88DV}{W} \tag{A2}$$

where

η propeller efficiency

W airplane weight, lb

D airplane drag, lb

V airspeed, mph

B brake horsepower, hp

If the indicated airspeed is assumed to be constant,

$$\frac{dR}{dB} = \frac{33000\eta}{W}\left(1 + \frac{B}{\eta}\frac{d\eta}{dB}\right) \tag{A3}$$

For the present results the values that were used are

$$\eta = 0.75$$

$$B = 950 \text{ hp}$$

$$W = 24{,}200 \text{ lb}$$

$$\frac{d\eta}{dB} \approx -0.0003 \text{ per hp}$$

and the correction to the rate of climb for power is

$$\Delta R \approx 0.6 \text{ ft/min/bhp} \tag{A4}$$

Correction for airplane weight.- From equation (A2) it can be shown that

$$\Delta R \approx -\frac{\Delta W}{W}\left[R + \frac{2(88)\left(D_iV\right)}{W}\right] \tag{A5}$$

where D_i is the induced drag. Using values for the present tests of

$$W = 24{,}200 \text{ lb}$$

$$V = 102 \text{ mph}$$

$$D_i = 1{,}260 \text{ lb}$$

$$R = 160 \text{ ft/min}$$

gives a correction to rate of climb for airplane weight of

$$\Delta R \approx -0.04 \text{ ft/min/lb} \tag{A6}$$

Correction for density altitude.- For the present test conditions at roughly constant indicated airspeed, dynamic pressure q may be assumed to be constant. From equation (A2) and the definition of q, the following relations may be obtained:

$$\frac{dR}{dh_d} = \frac{dR}{d\rho}\frac{d\rho}{dh_d} \tag{A7}$$

$$\frac{dR}{dh_d} = -88\frac{D}{W}\frac{dV}{d\rho}\frac{d\rho}{dh_d} \tag{A8}$$

and

$$\frac{dV}{d\rho} = -\frac{1}{2}\frac{V}{\rho}$$

where

ρ air density, slugs/cu ft

D airplane drag, lb

h_d density altitude, ft

For the average values of

$$V = 102 \text{ mph}$$

$$\rho = 0.002112 \text{ slug/cu ft}$$

$$\frac{d\rho}{dh_d} = -0.64 \times 10^{-7} \text{ slug-ft}^{-4}$$

$$D = 1{,}750 \text{ lb}$$

the correction to rate of climb for density altitude is

$$\Delta R \approx -10 \text{ ft/min/1000 ft density altitude} \tag{A9}$$

Correction for speed change.- It is assumed that the energy gained or lost by an airspeed change may be converted to potential energy or height and that the airplane rate of climb is constant with airspeed for the small airspeed changes being considered. Equating the change in potential energy to the change in kinetic energy gives

$$\Delta h \, g = \frac{\Delta\left(V^2\right)}{2} \tag{A10}$$

where Δh is the height change in feet. For a given change in airspeed, the equivalent height may be given approximately by

$$\Delta h \approx \frac{V \, \Delta V}{g}$$

or, in terms of rate of climb R,

$$\Delta R \approx \frac{V}{g} \frac{\Delta V}{\Delta t} \tag{A11}$$

In terms of indicated airspeed $\sigma^{1/2}V$, equation (A11) may be expressed as

$$\Delta R \approx \frac{\sigma^{1/2}V}{\sigma g} \frac{\Delta\left(\sigma^{1/2}V\right)}{\Delta t} \tag{A12}$$

For the present tests $\sigma^{1/2}V = 97$ mph and

$$\Delta R \approx 8 \text{ ft/min/mph change in indicated airspeed/min} \tag{A13}$$

APPENDIX B

THE STANDARD DEVIATION OF THE RATE OF CLIMB ATTRIBUTABLE TO TURBULENCE AND DURATION OF RUN

The following considerations appear to offer a physical basis for the form of the relation between the standard deviation of the rate of climb attributable to turbulence and run duration. Assume the following:

(1) The effect of turbulence on the rate of climb in a given unit of time is to add an increment of climb ΔR to the airplane rate of climb.

(2) The increment of climb ΔR is a random variable with a standard deviation σ_R. For a given run duration, the deviation of the rate of climb from a hypothetical still-air value is then given by

$$(\Delta R)_n = \frac{(\Delta R)_1 + (\Delta R)_2 + \ . \ . \ . + (\Delta R)_n}{n} \tag{B1}$$

where n is the number of time units and $(\Delta R)_i$ is the increment of climb for the ith unit of time.

The variance of the rate of climb for a number of runs of n units of time duration is then given by

$$(\sigma_R)_n^{\ 2} = \frac{1}{n^2} \sum_i^n (\sigma_R)_i^{\ 2} + \frac{1}{n^2} \sum_{\substack{i,j \\ (i \neq j)}}^n r_{ij} \ (\sigma_R)_i (\sigma_R)_j \tag{B2}$$

where r_{ij} is the coefficient of correlation between the rate of climb in the ith and jth time interval.

If $r_{ij} = 0$, equation (B2) reduces to

$$\left(\sigma_R\right)_n^{\ 2} = \frac{\left(\sigma_R\right)^2}{n}$$

or

$$\left(\sigma_R\right)_n = \frac{\sigma_R}{\sqrt{n}} \tag{B3}$$

Thus, the standard deviation of the rate of climb varies inversely with the square root of the duration of run.

The data of table IV were used to evaluate the coefficients of correlation r_{ij}. The results indicated that for the unit of time used (1 min) r_{ij} could be assumed to be equal to zero.

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TABLE I

AIRPLANE CHARACTERISTICS

Span, ft	95
Wing area, sq ft	987
Mean geometric chord, ft	10.4
Wing loading, lb/sq ft	25.3
Engine	Pratt and Whitney R-1830-92
Take-off power rating, hp	1,200
Normal maximum power rating, hp	1,050
Weight (average), lb	24,200 ± 300
Empty weight, lb	18,600

TABLE II

SUMMARY OF PERFORMANCE AND TURBULENCE DATA

(a) Forward center of gravity

Flight	Duration of run (min)	Rate of climb, R (ft/min)	Number of accelerations per minute greater than -						Turbulence class
			0.05g	0.1g	0.15g	0.2g	0.25g	0.3g	
9	4.98	123	18.8	2.8	0.2				II
	4.80	153	19.6	2.2	.4				II
	4.98	183	21.8	7.4	1.8	0.6			III
	4.88	185	18.2	2.6	.4	.2			II
	4.80	165	19.8	5.2	.8				III
10	4.90	145	21.8	8.0	2.0	.6			III
	4.85	141	23.4	25.4	1.0				III
	4.95	132	23.2	8.6	3.0				III
	4.85	113	19.4	9.0	4.0	2.0	0.4	0.2	III
11	4.65	143	11.2	.6	.2				II
	4.98	176	11.4	.4					II
	4.83	159	9.6	1.2					II
	4.80	164	8.2	.4					II
	4.72	119	6.8	.4					II
	4.93	172	5.8						I
	4.75	153	7.4	.6					II
12	4.88	181	.2						I
	4.93	137	1.2						I
	4.97	131	.6						I
	4.95	159	2.6	.4					II
	4.95	151	17.8	5.6	1.6	.4			III
13	4.75	167	1.0						I
	4.92	165	1.0						I
	4.97	169	1.6						I
	4.95	159	1.0						I
	4.90	154	1.6						I
	4.90	151	3.4	.2					II
14	4.97	165	25.2	8.0	1.8	.4			III
	4.88	181	25.6	15.2	5.2	.8	.2		III
	4.83	234	28.0	14.6	4.4	3.2	.2		III
	4.78	189	27.6	9.6	2.8	.4			III
	4.80	128	22.2	8.2	2.6	.2	.2		III
	4.87	189	18.6	5.2	1.0	.4	.2		III
15	4.93	157	.6						I
	4.83	145	1.2						I
	4.93	167	.4						I
	4.93	169	1.6						I
	4.92	159	1.0						I
16	4.98	109	13.0	2.0					II
	4.87	115	15.0	3.2	.2				II
	4.95	80	17.2	1.2	.2				II
	4.72	154	15.2	3.2	.2				II
	4.90	109	18.2	3.6	.6				II

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TABLE II - Concluded

SUMMARY OF PERFORMANCE AND TURBULENCE DATA - Concluded

(b) Rearward center of gravity

Flight	Duration of run (min)	Rate of climb, R (ft/min)	Number of accelerations per minute greater than -						Turbulence class
			0.05g	0.1g	0.15g	0.2g	0.25g	0.3g	
17	4.72	164	1.8						I
	4.85	177	1.0						I
	4.92	181	1.4						I
	4.85	168	3.0						I
	4.62	182	.4						I
18	5.07	175	1.8	.2					II
	4.97	174	.8						I
	4.98	180	.8						I
	4.95	167	1.4	.4					II
	4.90	181	1.8	.4					II
	4.70	175	.8						I
20	4.85	175	1.6	.2					II
	4.73	176	1.6						I
	5.00	188	.4						I
	5.00	160	1.0						I
	4.51	186	.8	.2					II
	5.00	187	1.6						I
21	4.90	150	13.0	1.4					II
	4.97	161	22.6	6.6	.6	.4			III
	4.90	168	26.4	7.0	1.8	1.0			III
	5.00	174	29.0	7.2	2.0	.8			III
	4.75	201	23.2	9.0	2.2				III
	4.90	165	13.2	2.2					II
22	4.97	141	25.0	5.0	1.8	.4	.2		III
	4.95	198	22.4	6.0	1.2				III
	5.00	199	21.2	6.6	2.4	.8	.4		III
	4.92	192	21.4	7.8	.8	.6	.2		III
	4.97	139	18.2	4.4	1.0	.4			II
	4.92	149	16.8	5.4	.4				III
23	4.92	224	25.2	10.4	2.0	.6			III
	5.02	140	25.0	9.2	1.2	.8	.2		III
	4.87	195	23.8	12.2	3.4	.6	.2	.2	III
	4.92	161	23.6	9.0	2.6	.4			III
	4.98	163	24.8	9.4	2.4	.6			III
	5.00	190	16.2	6.0	.4				III
24	4.98	163	1.6						I
	4.98	203	.2						I
	4.48	192	1.6						I
	5.00	197	1.0						I
	4.68	153	.6						I
	4.88	192	1.4						I
25	4.98	144	14.8	4.0					II
	4.72	137	10.0	6.2	3.0	2.2	1.0	.2	III
	4.97	161	26.6	14.4	6.6	2.4	.6	.4	III
	4.95	164	27.8	13.6	6.2	2.6	.2		III
	4.88	169	17.6	9.0	3.6	1.0	.2		III
	4.90	109	20.4	7.6	2.8	1.0	.6		III

TABLE III

STATISTICAL SUMMARY OF CLIMB PERFORMANCE TEST RESULTS

Turbulence class	Mean rate of climb, $\bar{R}$	Standard deviation of rate of climb, σ_R (*)	Standard deviation of mean rate of climb, $\sigma_{\bar{R}}$ (**)	Number of observations, N
Forward center of gravity				
I	159.4	13.9	3.7	14
II	140.8	28.5	7.1	16
III	162.8	32.6	9.0	13
Rearward center of gravity				
I	178.4	13.5	3.2	18
II	164.7	16.7	5.6	9
III	169.8	27.5	6.2	20

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$$* \ \sigma_R = \frac{\sqrt{\sum (R - \bar{R})^2}}{N - 1}$$

$$** \ \sigma_{\bar{R}} = \frac{\sigma_R}{\sqrt{N}}$$

TABLE IV

SUMMARY OF PERFORMANCE DATA FOR 1-MINUTE RUNS

(a) Forward center of gravity

Flight	Rate of climb (ft/min)				Turbulence class
	1st minute	2d minute	3d minute	4th minute	
9	142	171	67	95	II
	207	236	117	154	II
	187	171	220	93	III
	201	142	167	166	II
	153	177	158	138	III
10	145	155	105	217	III
	55	165	182	124	III
	217	56	130	168	III
	185	134	133	107	III
11	213	120	125	174	II
	116	127	188	164	II
	160	124	189	154	II
	176	168	158	156	II
	87	133	82	143	II
	198	185	149	103	I
	119	125	154	248	II
12	211	152	180	190	I
	114	132	178	175	I
	194	149	116	100	I
	115	199	180	138	II
	190	164	100	145	III

Flight	Rate of climb (ft/min)				Turbulence class
	1st minute	2d minute	3d minute	4th minute	
13	168	157	174	158	I
	164	166	165	167	I
	173	186	165	176	I
	191	158	155	185	I
	175	171	137	166	I
	151	75	157	158	II
14	400	188	-22	193	III
	226	310	152	93	III
	42	261	288	419	III
	170	177	347	104	III
	52	236	110	112	III
	321	203	146	122	III
15	186	191	169	149	I
	155	143	138	144	I
	168	146	191	189	I
	163	149	207	151	I
	135	163	173	148	I
16	77	156	60	50	II
	90	160	141	81	II
	20	124	77	81	II
	206	105	80	248	II
	68	138	106	133	II

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TABLE IV - Concluded

SUMMARY OF PERFORMANCE DATA FOR 1-MINUTE RUNS - Concluded

(b) Rearward center of gravity

Flight	Rate of climb (ft/min)				Turbulence class
	1st minute	2d minute	3d minute	4th minute	
17	187	175	180	144	I
	159	225	149	150	I
	201	195	171	138	I
	200	153	170	154	I
	200	211	182	160	I
18	161	204	138	176	II
	200	200	189	150	I
	174	181	180	159	I
	150	172	161	165	II
	201	204	157	155	II
	204	175	148	195	I
20	134	159	190	191	II
	162	185	207	128	I
	203	175	156	172	I
	162	100	167	130	I
	163	202	185	161	II
	175	196	154	176	I
21	157	213	91	188	II
	143	107	150	216	III
	146	142	204	145	III
	240	105	324	172	III
	264	123	356	179	III
	158	245	109	178	II
22	75	234	153	105	III
	144	172	249	167	III
	145	128	222	333	III
	39	289	145	231	III
	113	142	196	131	II
	161	57	150	166	III
23	210	231	219	204	III
	154	305	109	127	III
	85	184	176	251	III
	59	155	123	198	III
	108	202	231	180	III
	126	215	175	253	III
24	199	226	162	160	I
	220	247	220	145	I
	175	222	228	140	I
	192	164	221	196	I
	109	158	167	196	I
	174	171	184	211	I
25	59	181	206	168	II
	177	75	92	86	III
	101	198	137	235	III
	186	138	132	192	III
	135	176	188	269	III
	81	129	100	143	III

TABLE V

SUMMARY OF PERFORMANCE DATA BY RUN DURATION

(a) Forward center of gravity

Run duration (min)	Class I				Class II				Class III			
	Mean rate of climb, $\bar{R}$	Standard deviation of rate of climb, σ_R	Standard deviation of mean rate of climb, $\sigma_{\bar{R}}$	Number of observations, N	Mean rate of climb, $\bar{R}$	Standard deviation of rate of climb, σ_R	Standard deviation of mean rate of climb, $\sigma_{\bar{R}}$	Number of observations, N	Mean rate of climb, $\bar{R}$	Standard deviation of rate of climb, σ_R	Standard deviation of mean rate of climb, $\sigma_{\bar{R}}$	Number of observations, N
1	163.2	23.9	3.2	56	138.2	48.5	6.1	64	169.7	85.3	11.8	52
2	163.2	20.4	3.9	28	138.2	37.6	6.7	32	169.7	62.3	12.2	26
4	163.2	12.5	3.3	14	138.2	31.0	7.8	16	169.7	35.6	9.6	13

(b) Rearward center of gravity

Run duration (min)	Class I				Class II				Class III			
	Mean rate of climb, $\bar{R}$	Standard deviation of rate of climb, σ_R	Standard deviation of mean rate of climb, $\sigma_{\bar{R}}$	Number of observations, N	Mean rate of climb, $\bar{R}$	Standard deviation of rate of climb, σ_R	Standard deviation of mean rate of climb, $\sigma_{\bar{R}}$	Number of observations, N	Mean rate of climb, $\bar{R}$	Standard deviation of rate of climb, σ_R	Standard deviation of mean rate of climb, $\sigma_{\bar{R}}$	Number of observations, N
1	177.7	28.6	3.4	72	165.7	36.4	6.1	36	170.4	65.5	7.3	80
2	177.7	21.6	3.6	36	165.7	24.2	5.7	18	170.4	45.3	7.2	40
4	177.7	14.9	3.5	18	165.7	11.0	3.7	9	170.4	33.3	7.4	20

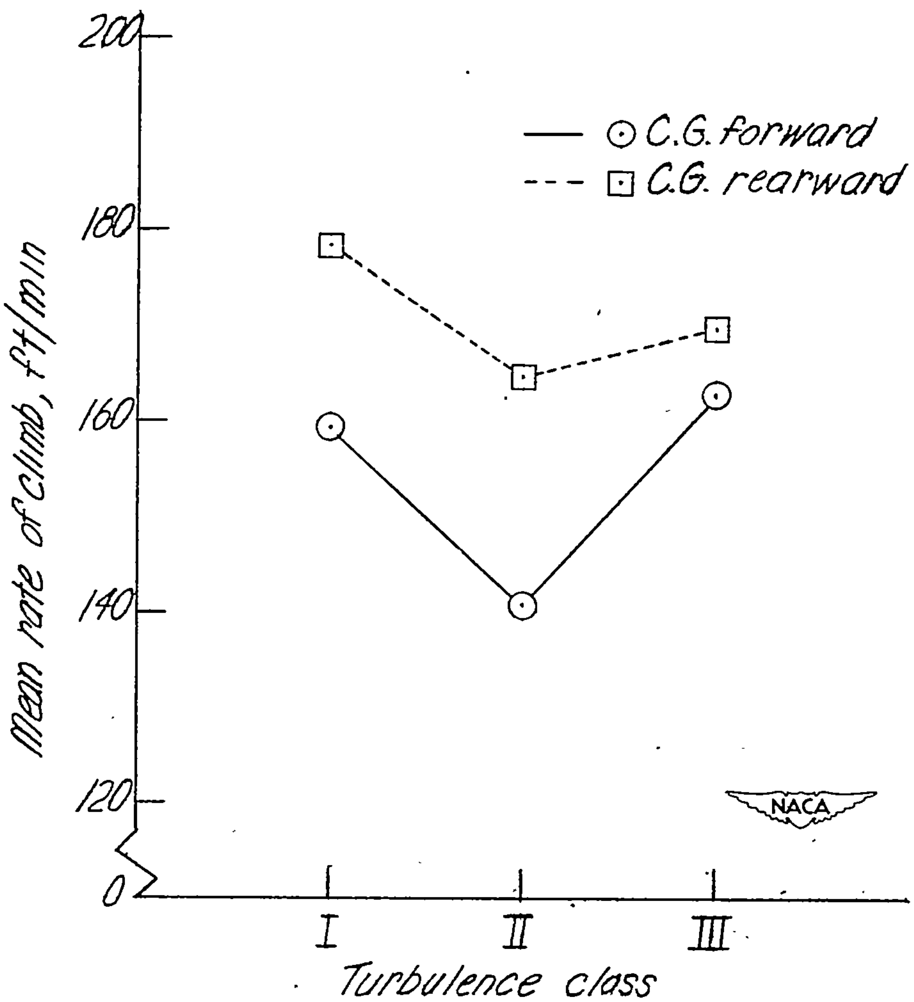


Figure 1.- Mean rate of climb as a function of turbulence class.

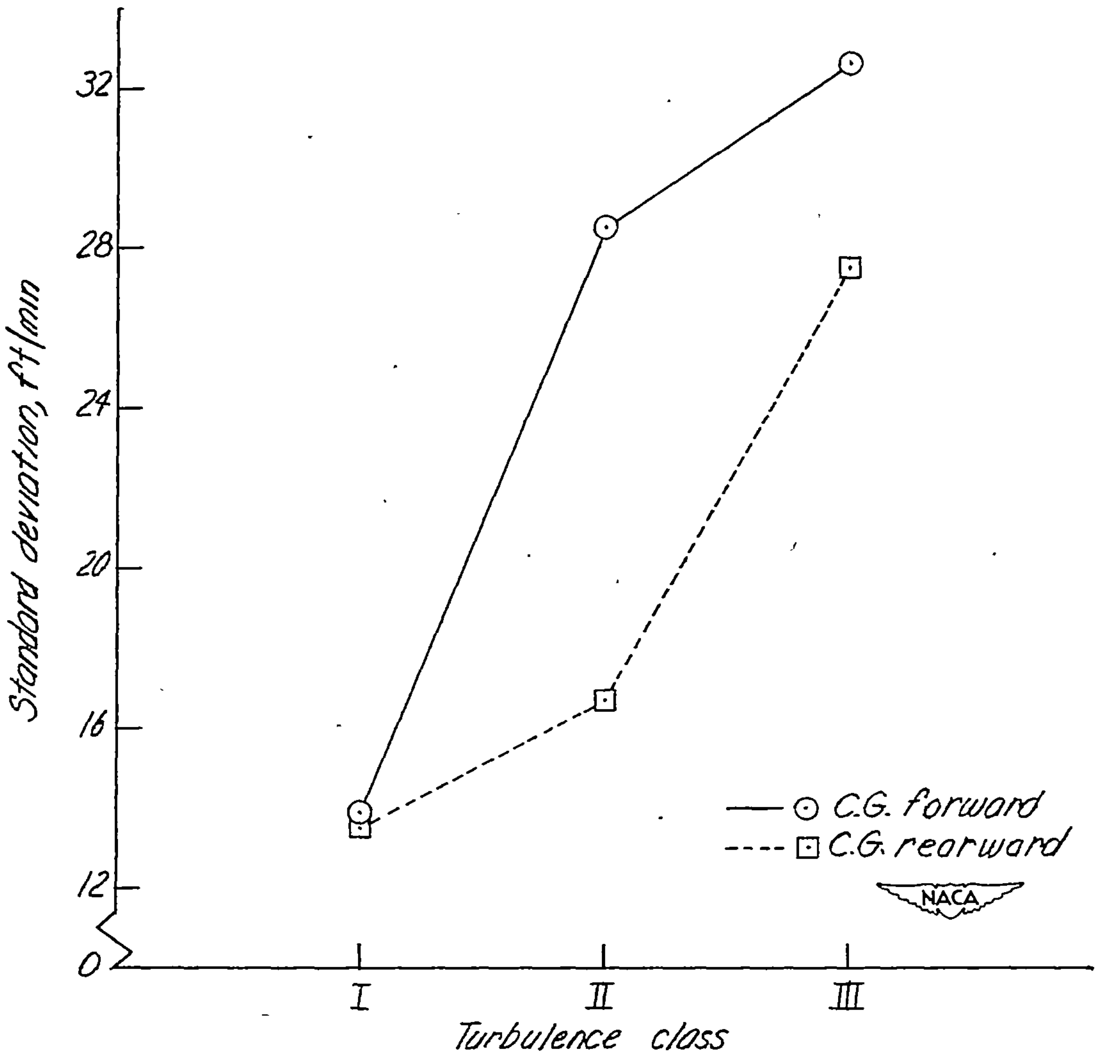
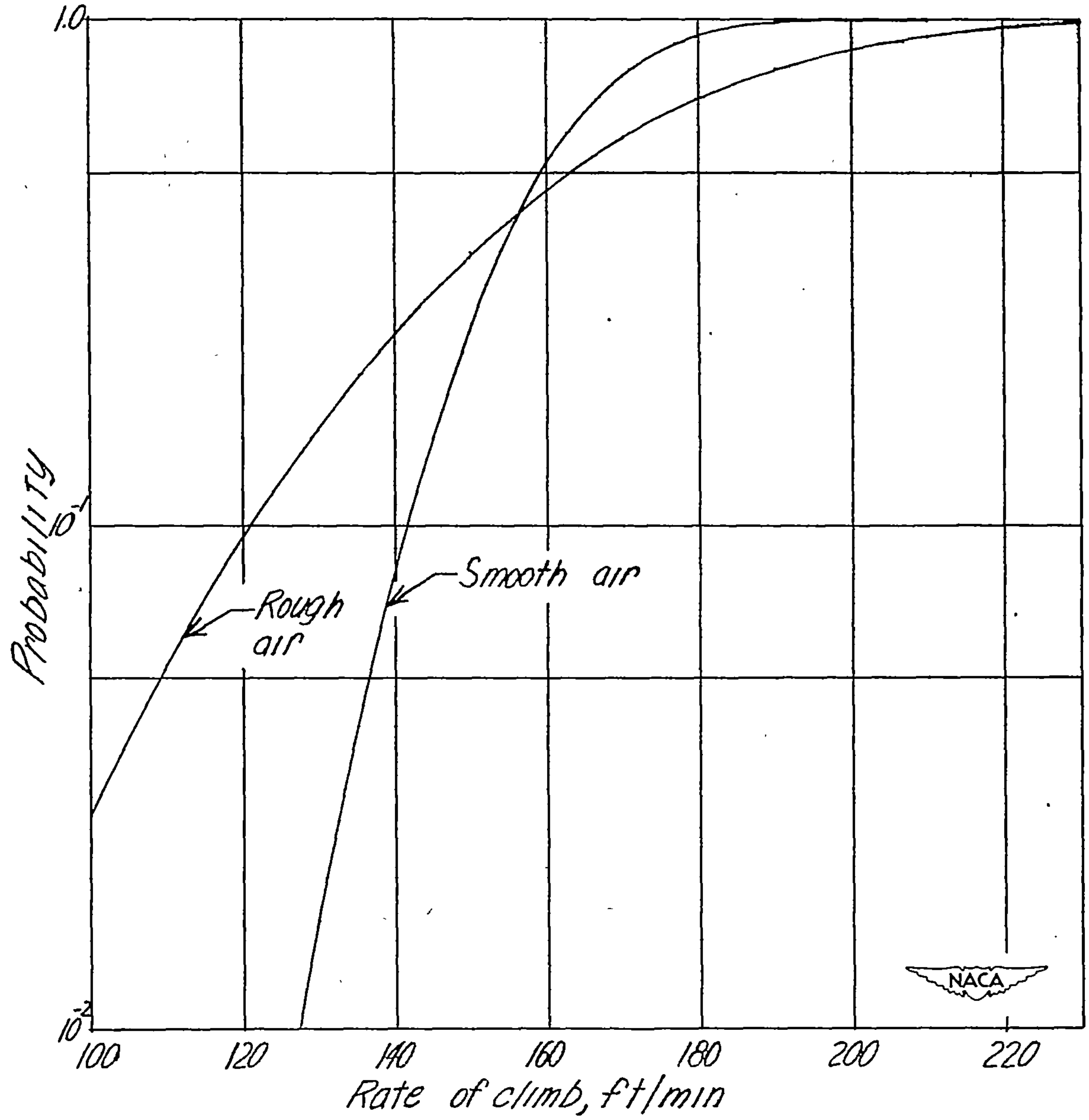
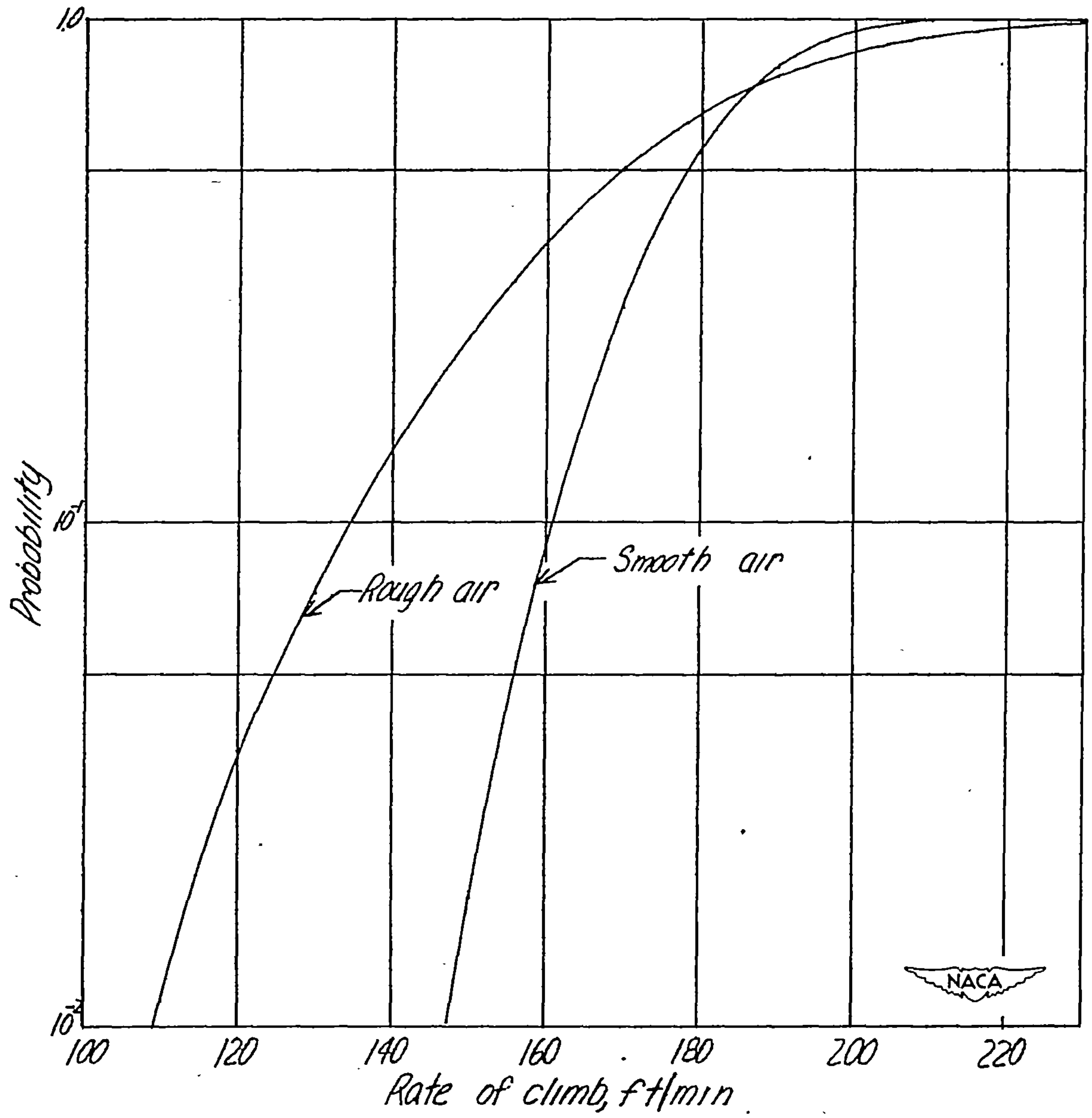


Figure 2.- Standard deviation of the rate of climb as a function of turbulence class.



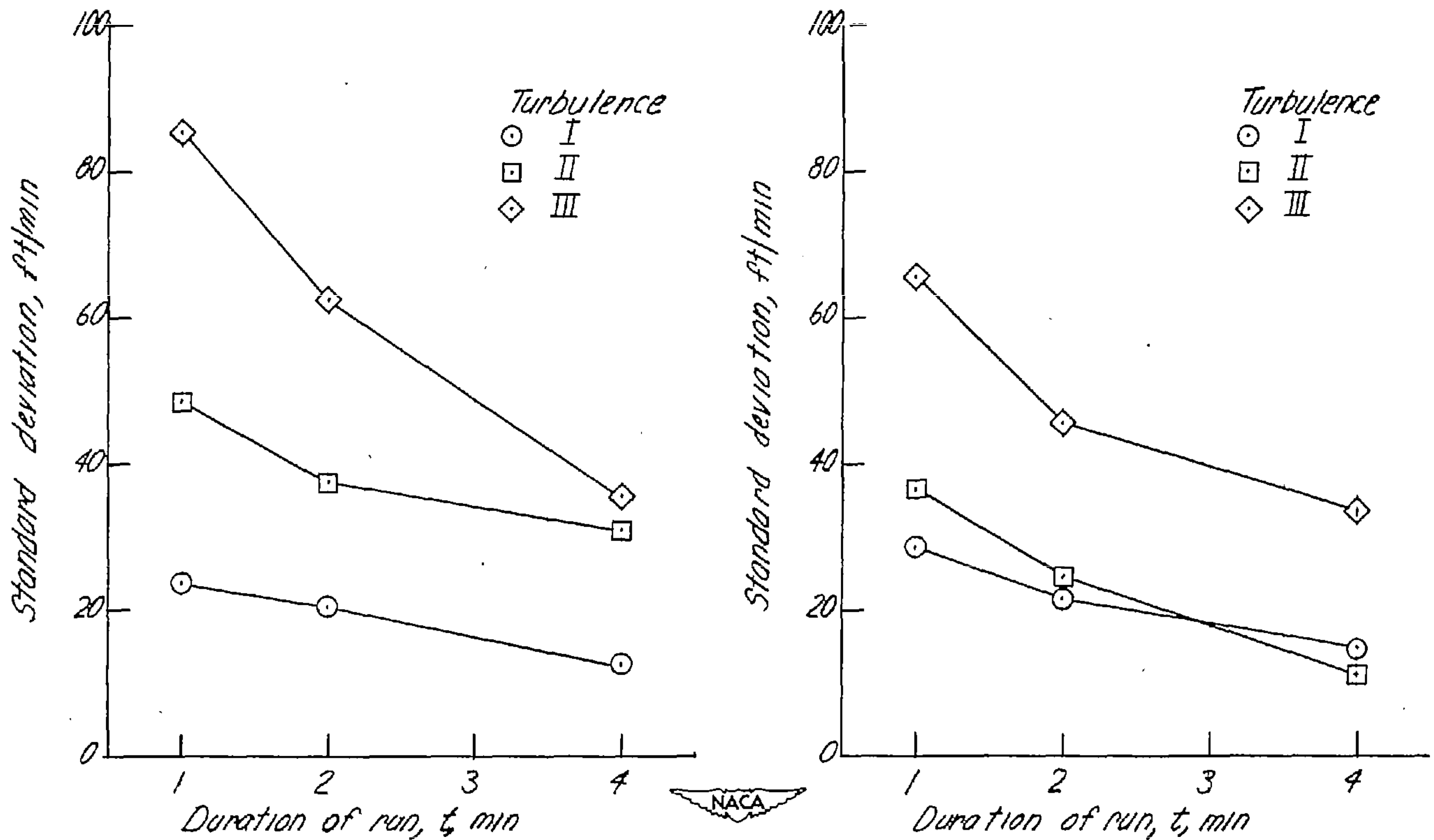
(a) Forward center of gravity.

Figure 3.- Probability of the rate of climb falling below the indicated values for smooth and rough air.



(b) Rearward center of gravity.

Figure 3.- Concluded.



(a) Forward center of gravity. (b) Rearward center of gravity.

Figure 4.- Standard deviation of the rate of climb as a function of run duration.

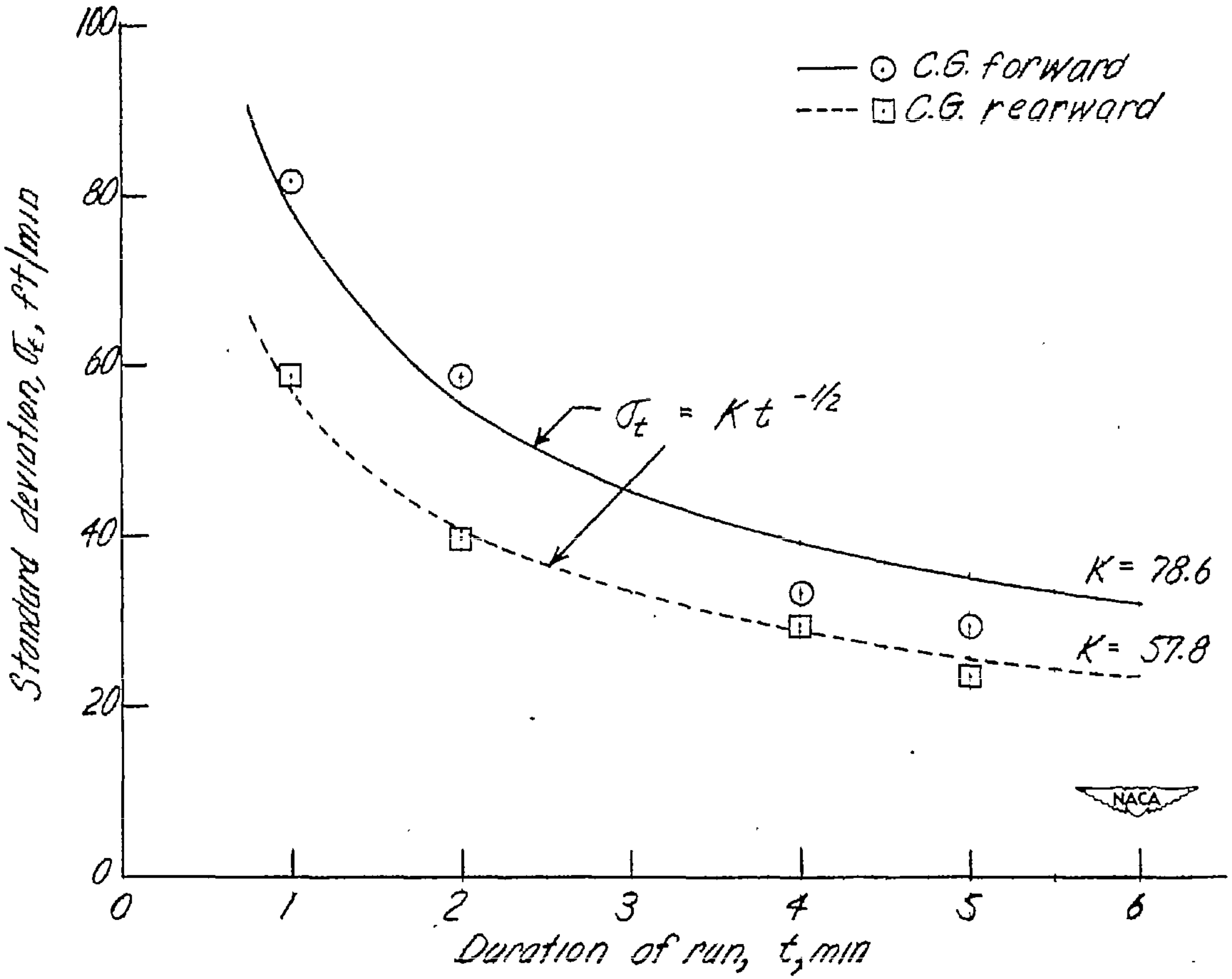


Figure 5.- Standard deviation of climb attributable to turbulence as a function of run duration.